Section 13.4

The Cross Product

We want a binary operation on two vectors $\vec{u}, \vec{v}$ that returns a vector that is orthogonal to both $\vec{u}$ and $\vec{v}$. There are infinitely many such vectors so we hope to find some natural choice.

Definition: Cross Product
Let $\vec{u} = u_1\vec{\imath} + u_2\vec{\jmath} + u_3\vec{k}, \vec{v} = v_1\vec{\imath} + v_2\vec{\jmath} + v_2\vec{k}$. The

$$\vec{u} \times \vec{v} = (u_2v_3 - u_3v_2)\vec{\imath} + (u_3v_1 - u_1v_3)\vec{\jmath} + (u_1v_2 - u_2v_1)\vec{k}$$

Theorem: Properties of the Cross Product
Let $\vec{u}, \vec{v}, \vec{w}$ be nonzero vectors in $\mathbb{R}^3$, and let a and b be scalars.

1. $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$
2. $(a\vec{u}) \times (b\vec{v}) = ab(\vec{u} \times \vec{v})$
3. $\vec{u} \times (\vec{v} + \vec{w}) = (\vec{u} \times \vec{v}) + (\vec{u} \times \vec{w})$
4. $(\vec{u} + \vec{v}) \times \vec{w} = (\vec{u} \times \vec{w}) + (\vec{v} \times \vec{w})$

Magnitude of Cross Product

We have now to find the magnitude and direction and relate this to physical phenomena to classify our definition as "natural."